7 Funktionen Loesungen

7.1 Trigonometrische Funktionen

1. Benötigt wird:

$$\begin{aligned} \sin x &= \sin(\pi - x) \\ \cos x &= -\cos(\pi - x) \\ \cos(-x) &= \cos(x) \\ \sin(-x) &= -\sin(x) \\ \tan x &= \frac{\sin x}{\cos x} \end{aligned}$$

a) $\sin \frac{2\pi}{3} = \sin(\pi - \frac{2\pi}{3}) = \sin(\frac{\pi}{3}) = \frac{1}{2}\sqrt{3}$

b) $\sin \frac{5\pi}{6} = \sin(\pi - \frac{5\pi}{6}) = \sin(\frac{\pi}{6}) = \frac{1}{2}\sqrt{1}$

c) $\sin \pi = \sin(\pi - \pi) = 0$

d) $\sin \frac{3\pi}{2} = \sin(\pi - \frac{3\pi}{2}) = \sin(-\frac{\pi}{2}) = -\sin(\frac{\pi}{2}) = -\frac{1}{2}\sqrt{4}$

e) $\sin \frac{11\pi}{6} = \sin(\pi - \frac{11\pi}{6}) = \sin(-\frac{5\pi}{6}) = -\sin(\pi - \frac{5\pi}{6}) = -\sin(\frac{\pi}{6}) = -\frac{1}{2}\sqrt{1}$

f) $\sin \frac{7\pi}{3} = \sin(\pi - \frac{7\pi}{3}) = \sin(-\frac{4\pi}{3}) = -\sin(\pi - \frac{4\pi}{3}) = -\sin(-\frac{1\pi}{3}) = \sin(\frac{\pi}{3}) = \frac{1}{2}\sqrt{3}$

g) $\sin \frac{29\pi}{6} = \sin(\pi - \frac{29\pi}{6}) = \sin(-\frac{23\pi}{6}) = -\sin(\pi - \frac{23\pi}{6}) = -\sin(-\frac{17\pi}{6}) = \sin(\pi - \frac{17\pi}{6}) = \sin(-\frac{11\pi}{6}) = -\sin(\pi - \frac{11\pi}{6}) = -\sin(-\frac{5\pi}{6}) = \sin(\pi - \frac{5\pi}{6}) = \sin(\frac{\pi}{6}) = \frac{1}{2}\sqrt{1}$

h) $\sin(-\frac{3\pi}{4}) = -\sin(\frac{3\pi}{4}) = -\sin(\pi - \frac{3\pi}{4}) = -\sin(\frac{\pi}{4}) = -\frac{1}{2}\sqrt{2}$

i) $\cos \frac{\pi}{6} = \frac{1}{2}\sqrt{3}$

j) $\cos \frac{\pi}{4} = \frac{1}{2}\sqrt{2}$

k) $\cos \frac{\pi}{3} = \frac{1}{2}\sqrt{1}$

l) $\cos \frac{\pi}{2} = \frac{1}{2}\sqrt{0}$

m) $\cos \frac{11\pi}{6} = -\cos(\pi - \frac{11\pi}{6}) = -\cos(\frac{5\pi}{6}) = cos(\frac{1\pi}{6}) = \frac{1}{2}\sqrt{3}$

n) $\cos \frac{3\pi}{4} = -\cos(\pi - \frac{3\pi}{4}) = -\cos(\frac{1\pi}{4}) = -\frac{1}{2}\sqrt{2}$

o) $\cos \frac{2\pi}{3} = -\cos(\pi - \frac{2\pi}{3}) = -\cos(\frac{2\pi}{3}) = -\frac{1}{2}\sqrt{1}$

p) $\cos \frac{4\pi}{6} = -\cos(\pi - \frac{2\pi}{3}) = -\cos(\frac{2\pi}{3}) = -\frac{1}{2}\sqrt{1}$

q) $\cos \frac{7\pi}{3} = -\cos(\pi - \frac{7\pi}{3}) = \cos(\pi - \frac{4\pi}{3}) = \cos(\frac{1\pi}{3}) = \frac{1}{2}\sqrt{2}$

r) $\cos -\frac{11\pi}{4} = \cos(\frac{11\pi}{4}) = -\cos(\pi - \frac{11\pi}{4}) = \cos(\pi - \frac{7\pi}{4}) = \cos -\frac{3\pi}{4}) = -\cos(-\frac{1\pi}{4}) = -\frac{1}{2}\sqrt{2}$

s) $\tan \frac{\pi}{6} = \frac{\sin \frac{\pi}{6}}{\cos \frac{\pi}{6}} = \frac{\frac{1}{2}\sqrt{1}}{\frac{1}{2}\sqrt{3}} = \frac{1}{\sqrt{3}})$

t) $\tan -\frac{\pi}{3} = \frac{\sin \frac{-\pi}{6}}{\cos \frac{-\pi}{6}} = \frac{-\sin \frac{\pi}{6}}{\cos \frac{\pi}{6}} = -\frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}\sqrt{1}} = -\sqrt{3}$

2.

α	β	a	b	c
$\frac{\pi}{4}$	$\frac{\pi}{4}$	1	1	$\sqrt{2}$
$\frac{\pi}{6}$	$\frac{\pi}{3}$	2	$\sqrt{12}$	4
kein Dreieck		$\frac{1}{2}\sqrt{3}$		$\frac{1}{2}$
$53,13°$	$26,87°$	4	3	5
$\frac{\pi}{6}$	$\frac{\pi}{3}$	1	$\sqrt{3}$	2
$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{4}{\sqrt{3}}$

3.

$$\begin{aligned}
\sin(4\alpha) &= 2(\sin(2\alpha)\cos(2\alpha)) \\
&= 4\sin(\alpha)\cos(\alpha)\cos(2\alpha) \\
&= 4\sin(\alpha)\cos(\alpha)(\cos^2(\alpha) - \sin^2(\alpha)) \\
&= 4\sin(\alpha)\cos^3(\alpha) - 4\sin^3(\alpha)\cos(\alpha) \\
&= 4(\sin(\alpha)\cos^3(\alpha) - \sin^3(\alpha)\cos(\alpha))
\end{aligned}$$

4.

$$\begin{aligned}
\cos(2\alpha) &= \cos(\alpha + \alpha) \\
&= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha \\
&= \cos^2\alpha - \sin^2\alpha \\
&= 2\cos^2\alpha - (\sin^2\alpha + \cos^2\alpha) \\
&= 2\cos^2\alpha - 1
\end{aligned}$$

5. Wenn die Seitenlänge des Würfels maximal sein soll, muss der Durchmesser der Kugel gleich der Diagnoalen des Würfels sein.

$$\begin{aligned} r &= 1 \text{ (Radius der Kugel)} \\ a &= \text{ (Kante des Wuerfels)} \\ D &= a\sqrt{3} \text{ (Diagonale des Wuerfels)} \end{aligned}$$

$$\begin{aligned} 2r &= a\sqrt{3} \\ a &= \frac{2}{\sqrt{3}} \end{aligned}$$

7.2 Exponentialfunktionen und Logarithmus

1. a) $1 = e^x \rightarrow x = 0$
   b) $8 = 2^x \rightarrow x = 3$
   c) $3 = 5e^x \rightarrow x = ln(\frac{3}{5})$
   d) $e = \frac{e^x}{e} \rightarrow x = 2$
   e) $9 = e^{cx} \rightarrow x = \frac{ln(9)}{c}$
   f) $3 = log_2(x) \rightarrow x = 8$
   g) $0 = log_{42}(x) \rightarrow x = 1$
   h) $0 = 5log_5(x) \rightarrow x = 1$
   i) $9 = 3ln(e^x) \rightarrow x = 3$
2. a) $lg2 + lg5 = lg10 = 1$
   b) $lg5 + lg6 - lg3 = lg10 = 1$
   c) $3lna + 5lnb - lnc = ln(\frac{a^3b^5}{c})$ für $c \neq 0$
   d) $2lnv - lnv = ln(v^2) - lnv = ln(v)$ für $v \neq 0$
   e) $\frac{1}{2}log_7 9 - \frac{1}{4}log_7 81 = log_7\sqrt{9} - log_7\sqrt[4]{81} = 0$
   f)

$$\begin{aligned} log_3(x-4) + log_3(x+4) &= 3 \\ log_3((x-4)(x+4)) &= 3 \\ log_3(x^2 - 16) &= 3 \\ x^2 - 16 &= 3^3 \\ x &= \pm\sqrt{43} \end{aligned}$$